BIBO Stability of A Class of Reset Control System

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Abstract

A reset element is a linear system whose states are reset to zero whenever its input meets a threshold. Such elements were introduced in feedback control systems with the aim of providing more favorable performance trade-offs than could be achieved by LTI control. This potential has been demonstrated in some recent experiments, but there is no guarantee of BIBO stability. This paper develops a sufficient condition for BIBO stability and applies it to an experimental reset control system of nontrivial complexity.

1 Problem Description

The structure of the reset control systems under consideration is shown in Figure 1, where \( L(s) \) denotes the linear plant and signals \( r(t), y(t), e(t), n(t) \) and \( d(t) \) represent reference input, output, error, sensor noise and disturbance, respectively. The first-order reset element (FORE), introduced in [1], is described by the first-order impulsive differential equation:

\[
\begin{align*}
\dot{x}_c(t) &= -bx_c(t) + e(t); \quad e(t) \neq 0 \\
x_c(t^+) &= 0; \quad e(t) = 0
\end{align*}
\]

where \( b \) is FORE’s pole and \( x_c(t) \) is its state. The time instants when \( e(t) = 0 \) are called reset times. We assume that these reset times \( t_i \) can be collected into the set

\[ I = \{t_i | e(t_i) = 0, \ t_i > t_{i-1} + \sigma, \ \sigma > 0, \ i = 1, 2, \ldots \}. \]

This definition implies that the interval between any two adjacent reset actions is greater than \( \sigma \). Assume that \( \{A, B, C\} \) is a minimal realization of \( L(s) \) and \( x_p(t) \in \mathbb{R}^n \) is the plant states. The state-space description of the reset control system in Figure 1 is

\[
\begin{align*}
\dot{x}_p(t) &= Ax_p(t) + Bx_c(t) \\
\dot{x}_c(t) &= -Cx_p(t) - bx_c(t) + w(t); \quad t \notin I \\
x_c(t^+) &= 0; \quad t \in I 
\end{align*}
\]

where \( w(t) = r(t) - n(t) - d(t) \) and \( y(t) = Cx_p(t) + d(t) \).

Figure 1: Block diagram of reset control system

2 BIBO Stability

Our main result is as follows:

**Theorem 1** Let \( A_{cl} = \begin{bmatrix} A & B \end{bmatrix} \). If there exists a \( \beta \in \mathbb{R} \) such that the transfer function

\[
h(s) = [\beta C (sI - A_{cl})^{-1}[0 \ldots 0 1]^T
\]

is strictly positive-real (SPR) and has no zero-pole cancellation, then system(2) is BIBO stable.

**Example:** We now apply the stability criteria in Theorem 1 to the experimental reset control system reported in [2]. This reset control design achieved good performance in a tape-speed control system, but there was no formal proof of stability. The structure of this control system is that shown in Figure 1 where the FORE’s pole is \( b = 30 \) and the linear plant transfer function \( L(s) \) is:

\[
\frac{5924839(s + 30)(s + 257)(s + 3.6)(s^2 + 20s + 216)}{s(s + 3 + 220)(s + 37)(s^2 + 14s + 62)(s^2 + 88s + 5925)(s^2 + 56s + 3679)(s^2 + 76.5s + 15790)(s^2 - 696s + 216100)(s^2 + 75s + 20030)(s^2 + 203s + 32800)(s^2 + 126s + 15790)}
\]

It is straightforward to show that the associated \( h(s) \) is SPR for \( \beta = 0.1 \). Hence, the experimental reset control system in [2] is verified to be BIBO stable.

References
